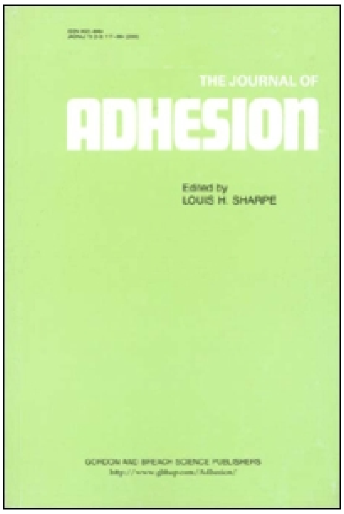

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Influence of Surface Properties on the Rebound of a Rigid Ball on a Rubber Surface

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It is well-known that molecular attraction forces, of the van der Waals type for rubber-like materials, act in the contact area between a polished and cleaned rigid ball and the smooth and flat surface of a rubber sheet; they increase the size of this contact area compared with the value which can be deduced from the classical theory of elasticity. Moreover, numerous experiments have shown that these forces are responsible for the existence of a kinetics of detachment in a pull-off test. In this study, we demonstrate that these long-range attractive forces also alter drastically the rebound of a rigid ball on a rubber surface. It is shown that during the movement of the ball out of contact with the rubber, work is done to peel the rubber off the ball surface and that this energy is the amount by which the surface adhesion effect reduces the resilience. A simple energy balance theory allows us to predict the rebound height and we show that there is a critical drop height below which the ball sticks and does not bounce at all. It is demonstrated that the simple energy balance theory proposed gives a correct prediction of the various observed rebound heights for several balls, when the critical drop height corresponding to a particular ball is known. In addition, this type of experiment provides us with a possible valuation of the thermodynamic work of adhesion (Dupré's energy of adhesion) and also allows us to determine the viscoelastic behaviour of the rubber-like material tested.

KEY WORDS: adherence; adhesion; contact area of a ball; dupré's energy of adhesion; elastomers; fracture mechanics concepts; rebound of a ball; rubber-like materials; viscoelastic behaviour

INTRODUCTION

In 1971, it was first shown[1] that rubber surfaces made optically smooth[2] can be brought into intimate contact with other surfaces equally smooth and, secondly, that the contact area between a rigid sphere and the flat and smooth surface of a rubber-like material is greater than the value which can be deduced from the classical theory of elastic contact,[3] because of the intervention of molecular attraction forces, of the van der Waals type for elastomers. In this work, Johnson *et al.*[1] used an energy balance theory in order to predict the correct size of the equilibrium contact between a spherical indenter and an elastic body as a function of the radius of this indenter, the normal applied load, the elastic modulus and the surface energy of the elastic solid. From this

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* One of a Collection of papers honoring Jacques Schultz, the recipient in February 1995 of *The Adhesion Society Award for Excellence in Adhesion Science, Sponsored by 3M.*

tremendous work, the kinetics of detachment in a pull-off test has been studied using fracture mechanics concepts in order to predict the contact duration in numerous constant unloading experiments.[4] Also, it has been observed that, under some circumstances, closely related to the intervention of these forces, a rigid ball strongly sticks against a glass-smooth rubber surface and does not rebound at all.[5]

Mainly, the understanding of the bouncing process of small particles on various substrates is important for numerous applications including aerosol physics and coatings technology for which a strong adhesion is wanted. In the literature, several experiments are described and theories are proposed to explain why small particles may be captured, so that no rebound occurs.[6-13] Particularly, these theories have attempted to combine both the surface effects and bulk dissipative deformation of solids.

The present paper reports numerous investigations made on ball rebound off rubber in the case where projectiles sizes are millimetric, *i.e.*, these are not small particles. A simple theory is proposed in order to predict conditions required to observe, or not, the rebound of polished steel balls and other projectiles from the flat and smooth surface of a soft natural rubber sample. This theory assumes only that the impact of a ball or projectile is described by the classical Hertz theory,[3] whereas the extraction from the rubber block follows the Johnson *et al.*[1] theory, as in a pull-off experiment, for instance. The very good agreement between experimental data and theoretical curves validates the simple theory proposed,[12] which is quite similar to the model independently proposed by Johnson and Pollock.[13]

ADHESIVE CONTACT OF A SPHERE

It is now well-known[1] that if a blunt, rigid asperity is normally brought near a smooth surface of a highly elastic solid, such as natural rubber, as soon as the distance becomes smaller than a few tenths of one μm, the two solids undergo molecular attraction of the van der Waals force type. These forces are predominant for distances greater than 2 nm (and they remain measurable for distances less than about 200 nm), and are the prime cause of the adhesion of solids, because stronger bonding forces are usually screened off by impurities and contaminants deposited or adsorbed on solid surfaces. When contact is made under an applied load, P, molecular attraction forces act in and around the area of contact and these forces are superimposed on the load P so that the contact seems to be maintained by an apparent normal load $P_1 > P$. The difference $(P_1 - P)$ represents an adhesive force and depends on the load P, on the contact geometry (shape and size) and on elastic and surface properties of the bodies in contact.

Our understanding of the adhesion of elastic solids made a marked advance with the introduction of the energy balance concept by Kendall[14] in 1971. This theory is based on optimizing the total energy (potential, elastic and surface energies) of the system at equilibrium, the elastic adherence force, *i.e.* the critical value of the tensile force P_{equi} at equilibrium, being derived by equating to zero the first derivative of the total energy. But this procedure cannot give information about the stability of the system at equilibrium, that depends on the second derivative of the total energy. This is the reason why Maugis and Barquins[4] have been led to reintroduce the concepts of fracture mechanics, such as the strain energy release rate, G, in order to study the stability by the sign of the derivative of G with regard to area of contact. This approach enables one to consider the edge of the contact area as a crack tip and to study the kinetics of crack propagation of this boundary *versus* time, as will be seen later.

Concerning the problem of the adhesive contact between a rigid sphere and the flat and smooth surface of a rubber-like material, the correct solution was found in 1971 by Johnson *et al.*[1] using an energy balance theory. The radius, a, of the contact area is calculated as a function of the load, P, the radius, R, of the rigid sphere, the elastic parameter, K, defined from Young's modulus, E, and Poisson's ratio, μ, of the elastic body such that $K = (4/3)E/(1-\mu^2)$, and the thermodynamic work of adhesion (Dupré's energy), w, defined from the surface (γ_i) and interface (γ_{ij}) energies of solids 1 and 2 contact by $w = \gamma_1 + \gamma_2 - \gamma_{12}$:

$$a^3 = PR/K + R(3\pi wR + (6\pi wRP + (3\pi wR)^2)^{1/2})/K \tag{1}$$

The second term on the right-hand side of the equilibrium relationship represents the correction to the classical Hertz theory[3] and it becomes predominant when the applied load, P, tends towards zero. For this particular contact geometry of sphere on flat, the apparent load, P_1, is given by:

$$P_1 = a^3K/R \tag{2}$$

and the normal elastic penetration, δ, of the spherical rigid punch is:

$$\delta = a^2/R - 2(a^2K - PR)/3aKR \tag{3}$$

the first term of the right-hand side corresponding to the Hertz's solution.

It has been shown[15] that the above relations can be directly found from the generalization by Sneddon[16] of Hertz's calculations to surfaces of any shape having axial symmetry, using Hankel's transformation and Abel's integral, if an integration constant, $\chi(1)$, is assumed to be non-zero. Commonly, for non-adhesive axisymmetric contacts, this constant is cancelled in order to have zero normal stresses at the edge of the contact area. This constant, $\chi(1)$, which depends on the shape of the axisymmetrical indenter, is proportional to the stress intensity factor, K_I, at the edge of the contact area, so that calculations of the normal stress and the discontinuity of the elastic displacement (penetration of the indenter) lead to formulae identical with those of fracture mechanics in the opening mode of crack propagation (Mode I). It, therefore, seems that the contact edge may be seen as a crack tip that advances or recedes if the normal applied load, P, is decreased or increased.

The edge of the contact area, like any three-dimensional crack, is subjected locally to a plane strain state so that the strain energy release rate, G, can be written:

$$G = (1/2)(1-\mu^2)K_I^2/E \tag{4}$$

where the factor (1/2) appears because the rigid indenter is not deformable. If P_1 represents the apparent load that, for a non-adhesive contact (*i.e.* if the Sneddon's integration constant $\chi(1)$ is zero), gives the same radius of contact area as under the load, P, if molecular attraction forces can act (*i.e.* $\chi(1) \neq 0$), then it was shown[15] that

$K_I = (P_1 - P)/(4\pi a^3)^{1/2}$, so that Equation (4) becomes:

$$G = (P_1 - P)^2/6\pi R P_1 \tag{5}$$

At equilibrium $G = w$, so that Equation (5) is the equilibrium relationship, where P_1 is given by Equation (2), as derived by Johnson *et al.*[1] on the basis of the energy balance theory. Figure 1a shows that, at the edge of the adhesive contact area under the normal applied load, P, the junction of the elastic solid to a rigid (glass) spherical indenter is vertical, as in the case of a half Griffith's crack, *i.e.* the geometry of fracture mechanics. For comparison, Figure 1b shows the profile corresponding to a non-adhesive contact, characterized by a tangential junction, under the Hertzian load $P_1 > P$ which is necessary to produce the same radius of contact area.

KINETICS OF ADHERENCE

The equilibrium state defined by $G = w$ (Figure 1a) may be disrupted by a change in normal applied load, P, or in elastic penetration, δ. If $G < w$, the contact area increases and the crack recedes, so that a healing phenomenon can be observed. Conversely, if $G > w$, the two solids begin to separate and the contact area decreases. So, it is important to point out now that these two behaviours, formation then rupture of contact, occur successively in the rebound of a rigid ball against the flat and smooth surface of a rubber block.

If the solids are separating ($G > w$), the difference $(G - w)$ represents the crack extension force applied to the edge of the contact area which can be seen as a crack tip. $(G - w)$ is the "motive" energy of the crack per unit of surface of the zone crossed by this

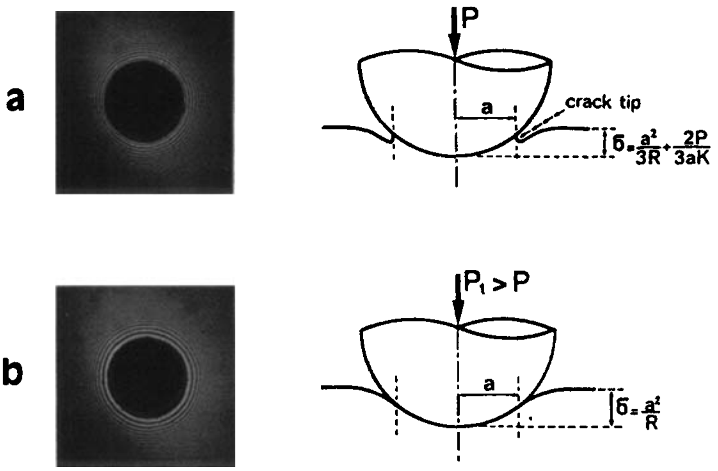

FIGURE 1 Newton's rings patterns photographed in normally-incident monochromatic light and corresponding profiles of the free surface at the immediate vicinity of the edge of contact between a glass ball and the flat and smooth surface of a natural rubber sample: (a) adhesive contact under the load, P; (b) non-adhesive contact under the Hertzian load $P_1 > P$ that produces the same radius of contact.

crack during its propagation. In a purely elastic solid, a crack subjected to a constant force $(G - w)$, per unit of length of the crack, continuously accelerates up to the Rayleigh wave speed. In a viscoelastic solid, such as a rubber-like material, the crack undergoes a viscous drag so that it takes on a limiting propagation speed, V, which is a function of the ambient temperature. On the assumption, confirmed by numerous experiments, that the viscoelastic losses are only localized at the crack tip, *i.e.* gross displacements are purely elastic, the general equation of the kinetics of adherence has been proposed:[4]

$$G - w \equiv w.\Phi(a_T.V) \tag{6}$$

where the left-hand side of the equation is the driving energy for the crack and the right-hand side is the viscous drag, which is proportional to Dupré's energy of adhesion, w, as first suggested by Gent and Schultz[17] in 1972. The main result is that the crack takes the propagation speed, V, so that corresponding viscoelastic losses exactly balance the driving energy $(G - w)$; this is the reason why the symbol "$\equiv$" is used in Equation (6). $\Phi(a_T.V)$ is a dimensionless function depending only on the crack propagation speed, V, and on the temperature, T, through the shift factor, a_T, of the Williams-Landel-Ferry[18] (WLF) transformation. The function $\Phi(a_T.V)$ is characteristic of the rubber-like material studied for the interfacial crack propagation in opening mode (Mode I) and may be directly linked to the frequency dependence of the imaginary component of Young's modulus.[19] Knowledge of the function $\Phi(a_T.V)$, by simple peeling experiments for instance, makes it possible to predict the change in contact area in all circumstances.

Numerous meticulous adherence experiments carried out with glass spheres, flat glass punches and flat-ended glass balls in contact with polyurethane PSM4 Vishay®, NR and SBR surfaces in pull-off tests at fixed load,[4,20] fixed grip conditions,[21] cyclic loading-unloading[22] (as in a repetitive push-on/pull-off test between two imposed loads), fixed crosshead velocity with the help of a tensile machine,[23] peeling from flat glass substrate of relaxed or stretched rubber sheets[4,24,25] (spontaneous delamination), and equilibrium contact geometry and rolling of glass, stainless steel and PMMA cylinders,[26-28] verify all the theoretical predictions.

Whatever the surface properties of the tested solids, or of such experimental parameters as geometry of contact, speed of separation, stiffness of testing machine, temperature, relative humidity, initial applied load which presses together the two solids and its duration of application, all results can be represented by a single master curve $\Phi(a_T.V) - a_T.V$ diagram. So, it has been shown[4] that over a large range of crack propagation speeds, V, the function $\Phi(a_T.V)$ varies as a power function of the speed:

$$\Phi(a_T.V) = k(T).V^n \tag{7}$$

where $k(T)$ is a parameter depending only on the temperature, T, and n is a characteristic parameter of the rubber-like material tested. For instance, n takes the value 0.60 for polyurethane[4] (PSM4 Vishay®), and $n = 0.55$ for a pure latex soft natural rubber sample,[26,28] using dicumyl peroxide vulcanization, as shown in Figure 2.

It is important to point out that the main interest of Equation (6) is that surface properties (w) and viscoelastic losses $(\Phi(a_T.V))$ are clearly decoupled from the loading conditions and the system geometry (rigid sphere-elastic flat surface), which only

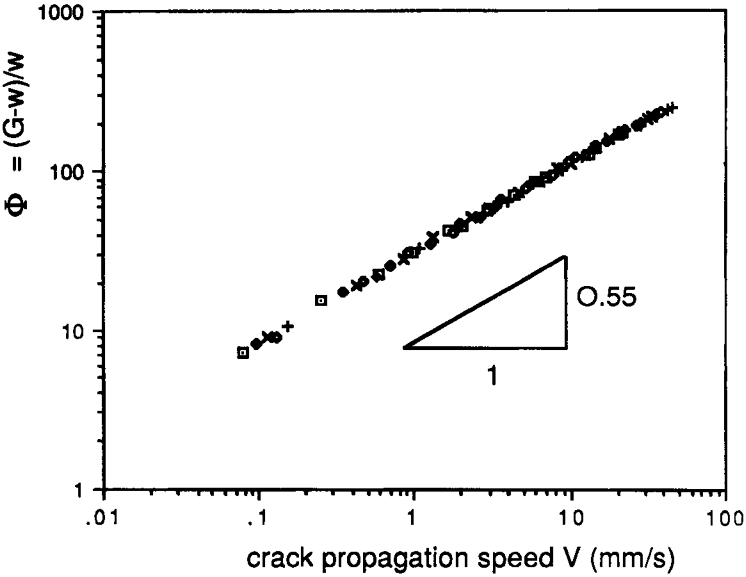

FIGURE 2 Master curve showing the dissipative dimensionless function $\Phi(a_T.V) = (G\text{-}w)/w$ of the soft natural rubber used, as a function of the crack propagation speed in opening mode at the interface with a rigid solid.[28]

appear in the strain energy release rate formula. Predictions assume only that: 1) the kinetic energy is negligible, *i.e.* the contact duration is adequate (not too short) to form an adhesive contact; 2) the rupture is adhesive, that is to say the propagation occurs at the interface of the two solids in contact so that experiments at equilibrium ($V = 0$) give Dupré's energy, w, of adhesion, and finally, 3) viscous losses are limited to the crack tip where strain rates are high, which implies that gross displacements are purely elastic and the strain energy release rate, G, can still be calculated by the Hertz theory during the crack propagation. All these conditions are satisfied for the contact between a rigid sphere and the flat surface of a rubber-like material block.

REBOUND OF RIGID BALLS

From concepts and experimental data outlined above, we have studied the rebound of a polished rigid ball (steel ball) on the flat and glass-smooth surface of a soft natural rubber sample.

Starting from the observation that there is a critical release altitude below which a rigid ball sticks on a rubber sample and does not rebound at all, numerous experiments were carried out to measure with accuracy this critical altitude and rebound heights when molecular attraction forces act. A simple energy balance theory is proposed to predict correctly the rebound height of several different-sized polished stainless steel balls, colliding with a horizontal, plane, glass-smooth and cleaned surface of a natural rubber thick sample.

The apparatus used for this study is schematically presented in Figure 3. A glass-smooth, polished steel ball was kept in position under a glass sheet with the help of a magnet. The vertical distance between the bottom of this ball and the cleaned surface of a thick (16 mm) and soft natural rubber block was measured accurately. Six steel balls were employed whose diameter and mass are given in Table I. In addition, four indenters were made from steel balls in order to obtain, on the one hand, three masses with the same diameter and, on the other hand, three radii of curvature with the same mass (Table I).

All experiments were initiated by gently raising the magnet from the glass sheet and rebound heights were recorded with the help of a video camera coupled to a video recorder in order to make accurate measurements. The "pure latex" natural rubber soft sample tested was obtained by moulding the compound against a glass-smooth polished steel plate, using the following vulcanization conditions: 2.0 parts of dicumyl peroxide and a temperature of 150 °C during 20 min, so that the glass transition temperature was equal to $-68\,^{\circ}\mathrm{C}$. The mechanical properties of the rubber specimen thus obtained were Young's modulus $E = 0.89$ MPa and Poisson's ratio $\mu = 0.5$, corresponding to the characteristic elasticity parameter K equal to 1.582 MPa.

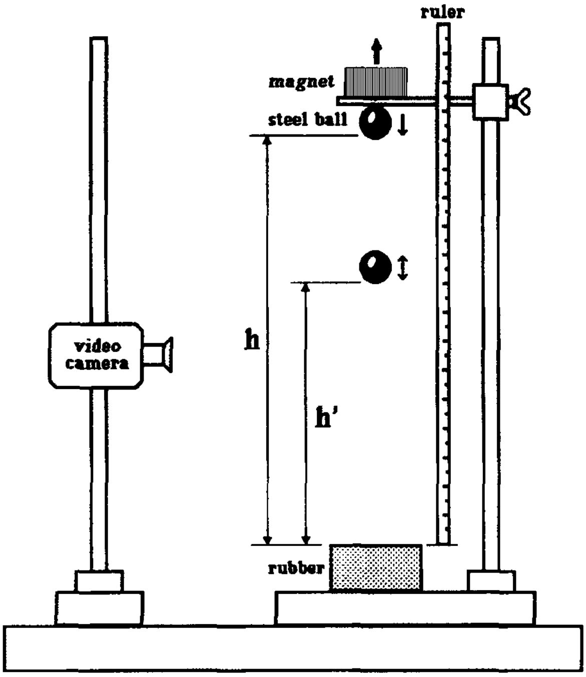

FIGURE 3 Schematic view of the apparatus used to study the rebound of a rigid ball against the surface of a rubber-like material. To prevent the action of van der Waals forces and to determine the resilience of the rubber sample, some experiments were carried out with rubber surfaces dusted with talcum powder.

TABLE I
Diameters and masses of the ten different indenters, balls and projectiles, used in the rebound experiments

Indenters	Diameter of curvature (mm)	Mass (*g*)
Balls		
No. 1	2	0.033
" 2	3	0.112
" 3	4	0.262
" 4	6	0.876
" 5	8	2.092
" 6	10	4.077
Projectiles		
No. 1	4	0.173
" 2	4	0.327
" 3	3	0.262
" 4	6	0.262

THEORETICAL CONSIDERATIONS

First, let us consider a rigid ball (of weight Mg) colliding with a non-adhesive plane (rubber surface contaminated with talcum powder to prevent adhesion of the ball) with a relative speed $v = (2gh)^{1/2}$. The ratio $\alpha_0 = h'/h$, where h (respectively, h') the release altitude (respectively rebound height), characterizes the losses in the bulk of the viscoelastic material, without intervention of molecular attraction forces. Due to the shortness of the collision time[29] the penetration of the ball into the rubber surface occurs according to the classical theory of Hertz, even for an adhesive surface, so that maximum values of radius, $a_{\max}$, of the contact area and the elastic penetration depth, $\delta_{\max}$, are closely related to the release altitude, h:

$$a_{\max} = \varepsilon h^{1/5} \tag{8}$$

$$\delta_{\max} = \kappa h^{2/5} \tag{9}$$

where $\varepsilon = ((5/2)\mathrm{Mg}\alpha_0^{1/2} R^2/K)^{1/5}$ and $\kappa = \varepsilon^2/R$, g being the acceleration due to gravity.

Assuming that the duration of penetration is equal to the duration of ejection from the rubber surface, each time it takes the value:[30]

$$\tau = \theta\delta/v \tag{10}$$

v being the drop speed at the impact instant ($v = (2gh)^{1/2}$) and θ the parameter equal to:

$$\theta = (3.29/2)(256/225)^{1/5}\alpha_0^{-1/5} \tag{11}$$

so that the ejection time of the ball can be rewritten:

$$\tau = \theta\kappa(2g)^{-1/2} h^{-1/10} \tag{12}$$

Assuming that molecular attraction forces act, it is obvious that they play an important role only during the ejection of the ball, due to the shortness of the collision time. The separation never occurs as a whole but the area of contact is progressively reduced until complete separation is achieved. This reduction in contact area can be considered as resulting from the propagation of a crack, and the concepts of fracture mechanics can be used. During its propagation, the crack undergoes a drag in proportion to w and is a function of the crack velocity, $V = -da/dt$, as already mentioned above.

The energy, W_{motor}, actually available when the maximum penetration depth, δ_{max}, given by Equation (9) occurs, is given by:

$$W_{\mathrm{motor}} = \alpha_0 Mgh \tag{13}$$

The energy dissipated by the crack propagation during the ejection of the ball can be written as kwV^n, where k and n are material dependent parameters (Equations (6) and (7)). This energy dissipated during the separation is the sum of two terms: the interfacial energy given by wS (where S is the contact area, $S = \pi a^2$) and the energy dissipated in the immediate vicinity of the crack, W_{drag}, given by the integration of the drag during the separation time. Unfortunately, the kinetics of the separation is unknown. Nevertheless, it is possible to assume (and confirm by further experiments), as in the Hertz theory,[30] that the ejection time is equal to the time of penetration, τ, and to take the ratio a_{max}/τ as an estimated average of the velocity $\langle V \rangle$ of the crack, so that:

$$\langle V \rangle = a_{\mathrm{max}} v/\delta 0 = (R(2g)^{1/2}/\varepsilon\theta) h^{3/10} \tag{14}$$

Taking into account Equations (6) and (7) rewritten using the estimated speed $\langle V \rangle$ of the crack, one obtains:

$$\langle (G - w)/w \rangle = k \langle V \rangle^n \tag{15}$$

Using the estimated average crack speed, $\langle V \rangle$, the interfacial dissipation term, W_{drag}, is given by:

$$W_{\mathrm{drag}} = kw\pi a_{\mathrm{max}}{}^2 \langle V \rangle^n = \pi\varepsilon^2 kw (R(2g)^{1/2}/\varepsilon\theta)^n h^{(4+3n)/10} \tag{16}$$

and scales as $kwS\langle V \rangle^n$. Due to the large value of the coefficient, k, $(k \gg 1)^{28}$, the interfacial energy, wS, is negligible compared with the interfacial dissipation term, W_{drag}, and the energy balance: $W_{\mathrm{motor}} = wS + W_{\mathrm{drag}}$ reduces to $W_{\mathrm{motor}} = W_{\mathrm{drag}}$. This balance gives the critical release altitude below which sticking occurs:

$$h_c = (\mathbf{A}/\mathbf{B})^{10/(6-3n)} \tag{17}$$

with:

$$\mathbf{A} = ((2/3.29)(225/256)^{1/5})^n (5/2)^{(2-n)/5} R^{(4+3n)/5} (2g)^{n/2} \pi kw$$

and

$$\mathbf{B} = (\mathbf{Mg})^{(3+n)/5} K^{(2-n)/5} \alpha_0{}^{(8-n)/10}$$

For $h > h_c$, the rebound height is obviously provided by:

$$h' = (W_{\mathrm{motor}} - W_{\mathrm{drag}})/Mg \tag{18}$$

when molecular attraction forces act. If the surface is dusted with talcum powder, for instance, the rebound height is commonly given by $h' = \alpha_0 h$, h being the release altitude.

EXPERIMENTAL RESULTS AND DISCUSSION

In a first set of experiments, six rigid and polished balls, made of steel (from ball bearings), were dropped onto the horizontal, plane, smooth, cleaned (with pure ethanol) and dried surface of the thick (16 mm) natural rubber sample (symbols and curves I in Fig. 4). The energy restitution coefficient, α_0, was determined with the two balls nos. 3 and 6 (Table I), when the surface was dusted with talcum powder to prevent adhesion of the balls (symbols and rectilinear curve II). It was calculated as $\alpha_0 = 0.625$, which is a value commonly found on a soft natural rubber sample when molecular attraction forces do not act.

Concerning heavy lines I, corresponding to a non-contaminated rubber surface, these are theoretical curves obtained by the following procedure. For each ball, numbered from 1 to 6 in Table I, the critical release altitude was accurately measured so that the mean value of the parameter $\langle k.w \rangle$, product of the temperature factor and the Dupré energy of adhesion, was correctly determined from Equation (17): $\langle k.w \rangle = 35.15\,\text{mJ.m}^{-2.55}.s^{0.55.}$ Taking into account the value $k = 1440\,\text{m}^{-0.55}.s^{0.55}$, previously assessed in similar ambient conditions,[28] one deduces that Dupré's energy of adhesion is equal to $w = 24\,\text{mJ.m}^{-2}$, a low value which is reasonable taking into account the shortness of the collision time.[29] So, if the ambient conditions (temperature

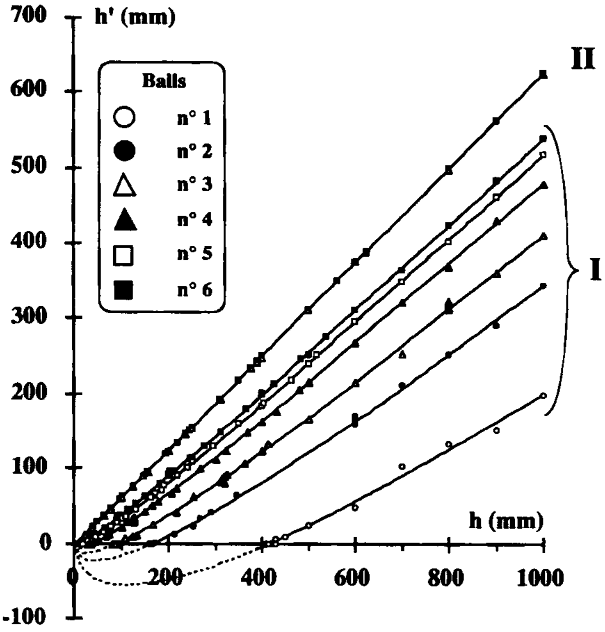

FIGURE 4 Rebound heights of six polished balls, made of stainless steel (each diameter and mass are listed in Table I), dropped on the horizontal, plane and smooth surface of a soft natural rubber sample (Young's modulus $E = 0.89$ MPa and Poisson's ratio $\mu = 0.5$). Curves I: the surface was cleaned with pure ethanol and dried with air, so that surface effects, due to van der Waals forces, were superimposed on bulk viscoelastic properties. Curve II: the surface was dusted with talcum powder to prevent adhesion. Experimental data (symbols) verify quite good computed predictions (heavy lines).

and relative humidity) are known, this rebound method offers the advantage of measuring the Dupré energy of adhesion for short contact time.

Starting from the experimentally-determined estimate of $\langle k.w \rangle$, the dissipated energy, W_{drag}, may be evaluated from Equation (16) and also the computed rebound height, from Equation (18), W_{motor} being computed from Equation (13). Figure 4 allows us to conclude that there is a very good agreement between the experimental data (symbols) and the rebound height predicted by the energy balance criterion proposed (heavy lines).

For balls made of the same material as these used in experiments previously described (Figure 4), the mass, M, is proportional to the cube of the radius of curvature, R, so that, in Equation (17), the ratio $\mathbf{A}/\mathbf{B}$ varies as R^{-1}. Hence, the critical release altitude, h_c, below which sticking occurs, varies as:

$$h_c \approx R^{-10/(6-3n)} \tag{19}$$

a negative power of R which is consistent with the recent theory of Johnson and Pollock.[13] Figure 5 shows h_c as a function of the ball diameter in log-log coordinates and the linear regression of all the points gives the slope $\psi = -2.28$, so that the value $n = 0.54$ can be deduced from Equation (19). There is a more accurate procedure to determine the parameter n, which explain the viscoelastic behaviour of the rubber-like material tested, as seen further on.

In a second set of experiments, the main goal was to predict the rebound height of several projectiles presenting a spherical cap made from a steel ball, as shown in Table I.

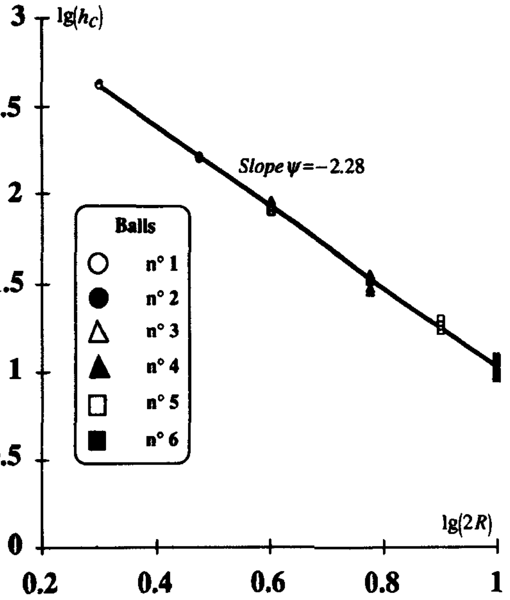

FIGURE 5 Master curve, in log-log coordinates, regrouping all the adhesive rebound critical heights measured with the six balls (data from Figure 4), which allows one to determine from the slope, ψ, of this rectilinear curve, the parameter n characterizing the viscoelastic behaviour of the natural rubber sample tested.

Figure 6 shows the ratio between the rebound height, h', and the corresponding release altitude, h, *versus* this release altitude for the same diameter of curvature (4 mm) but three different masses, the intermediate indenter being the steel ball no. 3 (Table I) as previously used. One can see that the agreement between experiment and theory is quite good, experimental points coinciding with computed curves drawn using the procedure already described. Obviously, all these curves tend towards the asymptotic value $h'/h = \alpha_0 = 0.625$, and it is clear that this asymptotic value is more rapidly reached with heavy loads, at constant diameter of curvature.

Conversely, Figure 7 shows the ratio between the rebound height h' and the corresponding release altitude, h, *versus* this release altitude for the same mass $M = 0.262\,g$ and three diameters of curvature, the intermediate indenter being the steel ball no. 3 (Table I) as in the previous rebound experiments. When the diameter of curvature increases at constant mass, the area of contact increases also, so that the prevalence of van der Waals forces increases and this is why the ratio h'/h, at imposed release altitude, h, decreases regularly with increasing diameter. Again, one can see that the agreement is quite good between experimental data and computed curves deduced from the simple energy balance theory proposed.

Equations (13) and (16) may be combined, so that Equation (18) can be written as:

$$Mgh' = \alpha_0 Mgh - Mg\pi\varepsilon^2 kw(R(2g)^{1/2}/\varepsilon\theta)^n h^{(4+3n)/10} \qquad (20)$$

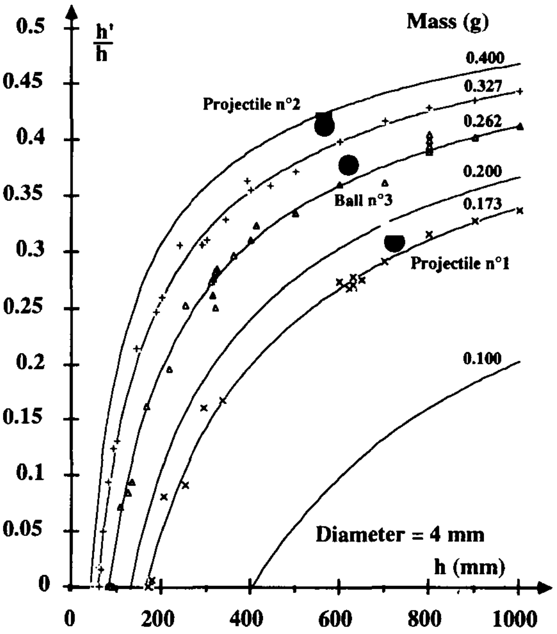

FIGURE 6 Rebound heights of two projectiles (nos. 1 and 2 in Table I), made of steel, and the ball no. 3, all presenting the same diameter of curvature (4mm), only the mass being changed. Experimental data (symbols) are in good agreement with corresponding predicted curves (heavy lines). Other computed curves have been drawn to demonstrate the behaviour if the load is changed at constant contact geometry.

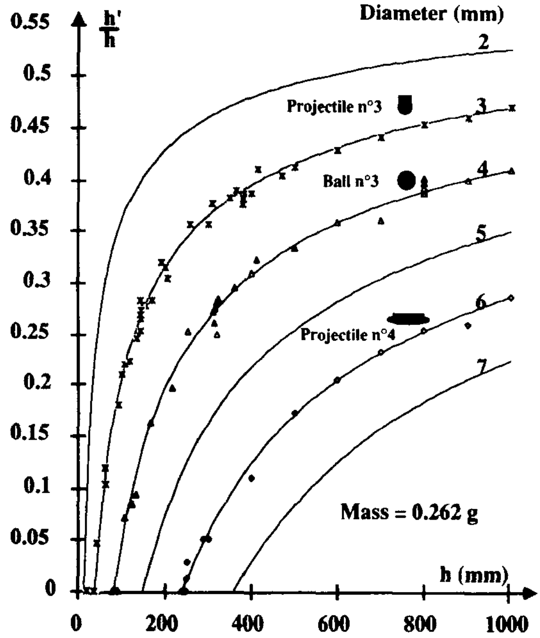

FIGURE 7 Rebound heights of two projectiles (nos. 3 and 4 in Table I), made of steel, and the ball no. 3, all presenting the same mass (0.262 g), only the diameter of curvature being changed. Experimental data (symbols) are in good agreement with corresponding predicted curves (heavy lines). Other computed curves have been drawn to exhibit clearly the behaviour if the contact geometry is changed at constant mass.

and when h is equal to the critical value h_c, then $h' = 0$, so that the previous Equation (20) becomes:

$$0 = \alpha_0 Mgh_c - Mg\pi\varepsilon^2 kw(R(2g)^{1/2}/\varepsilon\theta)^n h_c^{(4+3n)/10} \qquad (21)$$

So, it can be easily deduced from Equations (20) and (21):

$$Mgh' = \alpha_0 Mgh - \alpha_0 Mgh_c h^{(4+3n)/10} \qquad (22)$$

and to rewrite the Equation (22) as:

$$Mgh' = \alpha_0 Mgh - \Gamma h^{(4+3n)/10} \qquad (23)$$

where the parameter Γ regroups the geometry of the contact and elactic and superficial properties of the material tested. So, when the critical release altitude is assessed accurately, *i.e.* when $h = h_c$ or $h' = 0$, the previous Equation (21) allows us to express the value of the parameter Γ as a function of this critical release altitude, h_c:

$$\alpha_0 Mgh_c = \Gamma h_c^{(4+3n)/10} \qquad (24)$$

As a consequence, the authors think that the more important result of this study is to show that some rebound experiments, with a few different-sized polished balls colliding on clean surfaces of a rubber-like material, and also on dusted surfaces in order to measure the resilience, allows one to determine the exponent n

(Equation (7), which describes, as already written, the viscoelastic behaviour[4,19,31] of the material tested), n representing the power frequency variation of the imaginary part of the complex Young's modulus in a large range of frequencies or/and crack propagation speeds.

Indeed, from Equations (23) and (24), it can be written:

$$(\alpha_0 h - h')/h_c = \alpha_0 (h/h_c)^{\beta} \tag{25}$$

with:

$$\beta = (4 + 3n)/10 \tag{26}$$

So, from release altitudes, h, and corresponding rebound heights, h', critical rebound height values, h_c, and the resilience, α_0, of the rubber-like material tested, the slope, β, of the rectilinear curve representing Equation (25) allows one to calculate the parameter n which characterizes the viscoelastic behaviour. Figure 8 regroups all the experimental points measured on the cleaned, adhesive natural rubber surface (data of curve I of Fig. 4 and data of Fig. 6 and 7). Linear regression of all the points gives $\beta = 0.564$ so that, using Equation (26), n takes the value $n = 0.546$ which is in close agreement with the value $n = 0.55$ previously deduced from recent rolling experiments on the same rubber sample[28] (Fig. 2) and used to draw computed curves on Figures 4, 6 and 7. In fact, it is possible now to point out that the *a priori* knowledge of the exponent n was not necessary to draw computed curves on Figures 4, 6 and 7. The only required condition

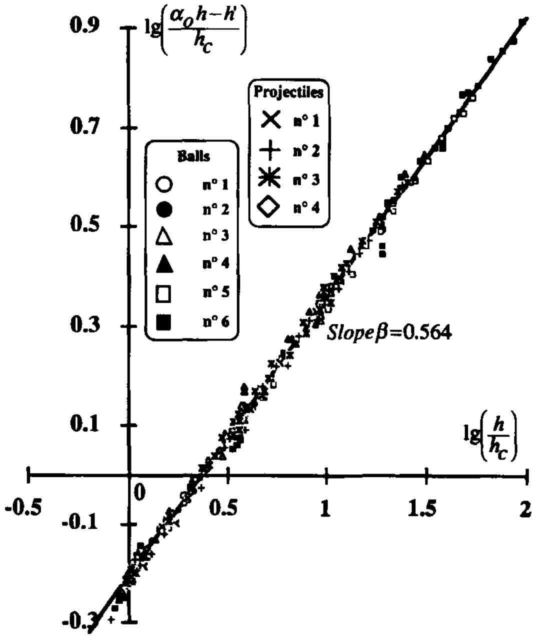

FIGURE 8 Master curve, in log-log coordinates, regrouping all the adhesive rebound critical heights measured (data from Figures 4, 6 and 7), which allows one to determine from the slope, β, of this rectilinear curve, the parameter n characterizing the viscoelastic behaviour of the natural rubber sample tested.

is that this exponent *n* exists in a large range of crack propagation speeds in opening mode.

CONCLUSION

The results of measurement of the rebound heights of steel balls and projectiles on the flat and smooth surface of a soft natural rubber sample and the simple energy balance theory proposed to confirm experimental data, allows one to conclude that: 1) the proposed theoretical model is valid; 2) this model permits one to evaluate the Dupré energy of adhesion for short contact time and most importantly 3) this rebound method allows a much better knowledge of the viscoelastic behaviour of rubber-like materials by the determination of the power law in crack propagation speed of the dissipative energy in the bulk.

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